Distance-Bounding Protocols:
Verification without Time and Location

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Abstract—Distance-bounding protocols are cryptographic protocols that securely establish an upper bound on the physical distance between the participants. Existing symbolic verification frameworks for distance-bounding protocols consider timestamps and the location of agents. In this work we introduce a causality-based characterization of secure distance-bounding that discards the notions of time and location. This allows us to verify the correctness of distance-bounding protocols with standard protocol verification tools. That is to say, we provide the first fully automated verification framework for distance-bounding protocols. By using our framework, we confirmed known vulnerabilities in a number of protocols and discovered unreported attacks against two recently published protocols.

Keywords—distance-bounding; security protocols; causality; formal verification; automatic verification

I. INTRODUCTION

Contactless systems are gaining more and more popularity nowadays. An increasing number of applications, including ticketing, access control, e-passports, tracking services, and mobile payments, make use of contactless communication technologies such as RFID and NFC. However, contactless communication is known to be vulnerable to relay attacks [1]: a man-in-the-middle attack where an adversary relays the verbatim messages that are being exchanged through the network.

Relay attacks are mostly used to break communication protocols with a bounded read range, such as smartcards (2-10 cm) or car keys (10-100 m). By simply relaying, an adversary is able to establish a long-range communication between two contactless tokens, which otherwise wouldn't be possible. This has been used, for example, by Francillon et al. [2] to break the passive keyless entry system of various modern cars.

To face relay attacks, Desmedt et al. [3], [4] introduced the notion of distance-bounding protocols, and the first such protocol was designed by Brands and Chaum [5]. Distance-bounding protocols use the round-trip time (RTT) of one or more challenge/response rounds to provide an upper bound on the prover-to-verifier distance (see Figure 1a). Through this scheme, security verification translates into the validity of the actual prover-to-verifier distance in comparison with the RTTs. More precisely, in a secure distance-bounding protocol, if the prover-to-verifier distance is $d$ and the RTT is $\Delta t$, then it must hold that $d \leq \frac{1}{2} \Delta t \cdot c$, where $c$ denotes the maximum network transmission speed (for radio-waves, this is the speed of light). This intuition is supported by the physical fact that no message can be transmitted at a speed higher than $c$.

In the context of distance-bounding protocols, their security has traditionally been assessed, over the years, by accounting for their resistance to three types of attack: mafia fraud [1], distance fraud [6], and terrorist fraud [6]. Resistance is measured in terms of probability of success of the adversary in a given adversary model [7]–[9]. However, this probabilistic analysis based on attack-resistance does not seem to be a promising verification scheme, as new attacks might be discovered in the future.

A clear and convincing proof of the flaws of the attack-based security analysis is the work by Cremers et al. [10]. In this work, the authors prove several protocols to be vulnerable to distance-hijacking attacks while they were previously considered secure as they resisted the then-existing attack types (mafia, distance and terrorist frauds). An important observation is that, like previous authors on distance bounding, Cremers et al. assumed a Dolev-Yao [11] adversary, so they did not introduce stronger adversary models to define their new type of attack.

Unfortunately, although the desired properties of a distance-bounding protocol can be precisely defined in current security models, it is not so straightforward to verify that a given protocol satisfies these properties. On the one hand, computational models [8], [12] typically lead to manual and complex security proofs. On the other hand, symbolic models [10], [13] rely on using adapted versions of higher-order theorem-proving tools such as Isabelle/HOL [14], which require a high degree of user intervention. This means that verifying the security of a distance-bounding protocol in the existing symbolic models requires not only a considerable amount of expertise, but also a significant time investment.

By comparison, well-established automated verification tools (such as Tamarin [15], ProVerif [16] and Scyther [17]) are able to verify traditional authentication properties in a straightforward and rapid way. These tools handle time as a discrete ordering of events, therefore verifying protocols with the notion of continuous time becomes difficult.

In this paper we argue that the notions of time and location are indeed not needed to specify and verify the security
of distance-bounding protocols. Surprisingly enough, such protocols can be verified by considering the causal order of events in protocol traces, similarly to authentication properties like aliveness and synchronization [18]. The intuition behind this observation is illustrated in Figure 1.

Figure 1a shows a regular challenge/response round, in which prover $P$ can only respond to verifier $V$’s challenge after having received the challenge. Therefore, $\frac{1}{2}c \cdot \Delta t'$ determines an upper bound on the distance $d$ between $V$ and $P$. Now, suppose that, due to a vulnerability of the protocol, $P$ is able to predict the appropriate response before having received the challenge (Figure 1b). This means that he will be able to send his response “too early”, leading to a shorter round-trip time $\Delta t' < \Delta t$ and thus to a smaller and incorrect distance calculated by $V$. Thus, if the protocol is insecure because $P$ can preempt the response, $P$ has sufficient knowledge to create the response before reception of the challenge. Now our main observation is that (assuming that there is no other causal relation between sending the challenge and $P$’s knowledge), $P$ could even have sent the response before $V$ sent the challenge (Figure 1c). From a causal point of view, this means that if there is a trace in which $P$ sends its response before $P$ receives the challenge, there must also be a trace in which $P$ sends the response before $V$ sends the challenge. Hence, a flaw in the protocol translates into such a wrongly ordered trace, which can be discovered through an analysis that does not consider time.

In the remainder of this paper, we will make this high-level intuition precise in the following way:

- First we introduce a security model, based on Basin et al. [13], [19] and formally define the notion of secure distance-bounding using time and location (Section III).
- Then, in Section IV, we analyse the semantic domain and formulate a number of basic properties that provide a sufficient characterization of the semantics to prove our main result. The purpose of this step is to make our result independent of the particular time/location semantics used.
- Next, we formulate our notion of causality-based secure distance-bounding, which does not refer to time and location, and we prove it equivalent to the previously defined notion of secure distance-bounding (Section V).
- In order to validate our results, we demonstrate an implementation of causality-based secure distance-bounding in Tamarin [15] and use it to perform a large-scale analysis of published protocols (Section VI). Our analysis results coincide with previous formal analyses, such as the report by Cremers et al. [10]. In addition, we uncover previously unreported vulnerabilities on recently published protocols.

II. BACKGROUND

Distance-Bounding Protocols: The first distance-bounding protocol was designed by Brands and Chaum [5] and it is composed of three phases. The slow phase (a.k.a. initial phase, setup phase) is where the parties agree on the parameters of the session, such as nonces. Then the fast phase (a.k.a. critical phase, distance-bounding phase, timed phase) is executed, consisting of a number of challenge/responses rounds, where the verifier measures the round-trip times. Finally, a verification phase (a.k.a. final phase, authentication phase) takes place, in which the verifier makes a decision on whether the prover successfully passed the protocol. This is done by checking the correctness of all round-trip times and the prover’s proof of knowledge of a valid signature.

Another well-known distance-bounding protocol was proposed by Hancke and Kuhn in [20]. An abstraction of this protocol is shown in Figure 2. The first two messages compose the initial phase of the protocol, where the verifier $V$ sends his nonce $N_V$ to the prover $P$ who replies back with his nonce $N_P$. Then the fast phase starts (represented by dashed arrows) with $V$ sending his challenge $C$ to $P$ whose response is $h(k, N_V, N_P, C)$, where $k$ is the shared secret key between $V$ and $P$ and $h$ is an irreversible cryptographic function. The verification phase is represented by $V$’s claim that “$P$ is close”. The protocol seems to be secure, as for an attacker (who could be an untrusted prover) to pass the protocol, he must know either the verifier’s challenge in advance or the shared secret key between the verifier and the intended prover. However, due to the particular choice of $h$, a mafia-fraud attacker successfully passes the protocol with a non-negligible probability of $(3/4)^{|C|}$ (see [20] for further details).

One of the main differences between Brands and Chaum’s protocol and Hancke and Kuhn’s protocol is the following. In the former, the fast phase messages do not rely on long-term secret keys whereas in the latter protocol, such a reliance does exist. Various protocols have been proposed following this characteristic of Brands and Chaum’s approach, e.g. [21]–
Attacks on Distance-Bounding Protocols: Although distance-bounding protocols solved the problem of relay attacks to some extent, more sophisticated attacks have emerged, such as mafia fraud, distance fraud, terrorist fraud and distance hijacking. Mafia-fraud attacks were introduced in [1], in which a dishonest agent \( I \) uses an honest prover \( P \) to provide a verifier \( V \) with a false upper bound on the distance between \( V \) and \( P \). Some authors consider mafia-fraud attacks to be the same as relay attacks. Others, however, classify mafia-fraud attackers stronger than relay attackers by assuming that the former can manipulate/modify the messages, rather than simply relaying them.

A distance-fraud attacker [6] is a dishonest prover \( I \) whose goal is to provide a verifier \( V \) with a false upper bound on \( V \)'s distance to \( I \). In particular, for this type of attack, \( I \) does not use any other prover to perform his attack.

More sophisticated attacks are terrorist fraud and distance hijacking. Terrorist-fraud attacks were first discussed in [6] in which the attacker prover \( I \) cheats on the upper bound on the distance between a verifier \( V \) and a dishonest prover \( P \), without learning \( P \)'s secret key material. Distance hijacking was introduced by Cremers at al. in [10], in which a dishonest prover \( I \) makes use of honest provers in order to provide a verifier \( V \) with a false upper bound on the distance between \( V \) and \( I \).

Probabilistic Security Analysis: The work by Avoine et al. [7] introduces a framework that explores the adversary’s capabilities and strategies and the influence of provers’ abilities to tamper with their devices. New concepts in the distance-bounding field are introduced such as black-box and white-box models.

The concepts sketched in [7] were soon formulated in computational models. For example, Dürholz et al. formalized the classical attacks (except for distance hijacking) by using an adversary model that does not allow for corrupted verifiers [8]. Bourreau, Mitrokotsa, and Vaudenay introduced a more general model [12] by allowing adversaries to interact with multiple provers and verifiers, hence capturing distance hijacking [10].

Mauw, Toro-Pozo, and Trujillo-Rasua [28], [31] developed a probabilistic analysis of the security of a class of distance-bounding protocols in terms of mafia fraud. This class includes distance-bounding protocols that do not have a final verification phase and are based on precomputation (e.g. [20], [26]–[28], [30], [32]–[34]). They proposed a set-of-automata representation of protocols that allows the analyst to generically compute the success probability of mafia-fraud attacks.

Symbolic Security Analysis: Meadows et al. [23] proposed a formal framework to verify distance-bounding protocols. Their approach does not particularly deal with multi-prover scenarios, therefore neither distance-hijacking nor terrorist-fraud attacks would be detected.

The first formal framework for distance-bounding protocols with multi-prover scenarios was proposed by Malladi et al. [35], along with a software tool. They analyse the signature-based Brands and Chaum’s protocol and find an attack in which an adversary who is not in the vicinity of the verifier still passes the protocol. They call this attack the farther adversary scenario. Moreover, to solve the security issue they found, they observed that including the prover’s identity in the signature would make the protocol no longer vulnerable to farther adversary attacks.

Basin et al. [13], [19] introduced a simple yet powerful formal approach for distance-bounding verification. Their model captures dishonest prover behaviors and, by extension, distance-fraud and distance-hijacking attacks, of which the latter was referred to as impersonation attacks. Their implementation of the formalization is written in the higher-order logic theorem prover Isabelle/HOL [14]. Similarly to Malladi et al. [35], they prove that the signature-based Brands and Chaum’s protocol can be fixed by explicitly adding the prover’s identity to the responses in the fast phase.

In [10], Cremers et al. extended Basin et al’s model to capture bit-level message manipulation on wireless networks, introduced as overshadowing in [36]. Supported by this, they proved that including the prover’s identity (neither by XORing it with the challenge responses nor by using secure channels) in Brands and Chaum’s protocol does not solve its vulnerability to distance hijacking.

It is still an open problem how to model terrorist fraud in a symbolic security model. Originally, a terrorist-fraud attack consisted in a far-away dishonest prover passing the distance-bounding protocol with the help of the adversary, but without leaking the prover’s long-term key [6]. Many attempts to formalize this intuition have been made in computational models [8], [24], [37], [38]. Yet, there seems to be no agreement on the appropriate definition.

III. A SECURITY MODEL BASED ON TIME AND LOCATION

In this section we describe the formalism of Basin et al. [13], [19] which is the basis for our work. The formalism employs logic theories to handle inductively-defined sets of traces that represent the protocol’s executions. It considers execution traces that consist of a sequence of timed-events, e.g. denoting...
the sending and reception of messages, where the timestamps represent the point in time at which the events occurred.

**Agents and Messages:** Participants in a protocol execution are called *agents*. The set of agents is denoted by Agent, and \{Honest, Dishonest\} is a partition of the set of agents into honest and dishonest agents.

During a protocol execution, agents exchange messages through the network. Basic messages are agent names (Agent), nonces (Nonce), and constants (Const). More complex messages can be defined by using atomic messages as the arguments of a function, by pairing them together into a single message or by denoting an encrypted message. Formally, the set of messages \( \text{Msg} \) is defined by the following grammar, where \( \text{atom} \in \text{Const} \cup \text{Agent} \cup \text{Nonce} \) and \( f \in \mathcal{F} \) are terminal symbols and \( \mathcal{F} \) is a countably infinite set of function symbols.

\[
\text{Msg} ::= \text{atom} \mid (\text{Msg}, \text{Msg}) \mid \{\text{Msg}\}_\text{Msg} \mid f(\text{Msg}).
\]

The term \( (m_1, m_2) \) denotes the pairing of messages \( m_1 \) and \( m_2 \). Further, \( \{m_1\}_2 \) stands for the encryption of \( m_1 \) with the key \( m_2 \). An agent’s signature on a message is represented by the encryption of the message with the secret key of the agent. Finally, \( f(m_1) \) indicates the output of the function \( f \) on the input \( m_1 \). Functions with multiple arguments can be represented through pairing of arguments.

Agents’ cryptographic keys are denoted by the functions \( pk: \text{Agent} \to \text{Msg}, sk: \text{Agent} \to \text{Msg} \) and \( sh: \text{Agent} \times \text{Agent} \to \text{Msg} \) that indicate the asymmetric public key of an agent, asymmetric secret key of an agent and the symmetric shared key of two agents, respectively. Lastly, the function \( -1: \text{Msg} \to \text{Msg} \) maps an encryption key onto the corresponding decryption key, and vice-versa.

The set \( \mathcal{B} = \{sk, pk, sh, -1\} \subseteq \mathcal{F} \) is the set of basic functions and its functions are assumed to satisfy that \( sh(A, B) = sh(B, A), pk(A)^{-1} = sk(A) \) and \( sk(A)^{-1} = pk(A) \); for all \( A, B \in \text{Agent} \). In addition, we assume that \( k \notin \{pk(A), sk(A)\} \) implies \( k^{-1} = k \); for all \( k \in \text{Msg} \) and \( A \in \text{Agent} \). These assumptions represent the properties for symmetric and asymmetric encryption/decryption.

**Events and Traces:** An event denotes an agent’s action, such as sending or receiving a message, or an agent’s security claim. We define the set of events \( \mathcal{Ev} \) via the following grammar, for \( A, B \in \text{Agent} \).

\[
\mathcal{Ev} ::= \text{send}_A(\text{Msg}) | \text{recv}_A(\text{Msg}) | \text{claim}_A(B, Ev, Ev).
\]

Given messages \( m_1 \) and \( m_2 \), and agents \( A \) and \( B \), \( \text{send}_A(m_1) | m_2 \) indicates that \( A \) has sent the message \( m_1 \) and updated the agent’s local state with the message \( m_2 \), and \( \text{recv}_A(m_1) \) means that \( A \) has received \( m_1 \). In the original model, claiming events have the form \( \text{claim}_A(B, d) \), where \( d \in \mathbb{R} \) is a distance value. This allows an agent \( A \) to claim that another agent \( B \) is within a radius of length \( d \), which is computed based on the round-trip time of a message exchange. We will make the message exchange explicit, and use \( \text{claim}_A(B, e_1, e_2) \) where \( e_1 \) and \( e_2 \) are the events used to compute the round-trip time and, by extension, the distance bound \( d \).

We define the sets \( \text{Send}, \text{Recv} \subseteq \mathcal{Ev} \) of all send and receive events, respectively. The function \( \text{actor}: \mathcal{Ev} \to \text{Agent} \) maps events onto their corresponding actor agent (i.e., the instance of \( A \) from the syntax). We extend this notation by using \( \text{actor}(\alpha) \), for a given trace \( \alpha \), to refer to the set \( \{\text{actor}(e) | (t,e) \in \alpha\} \). We require for an event \( \text{claim}_A(B, e_1, e_2) \) that \( \text{actor}(e_1) = \text{actor}(e_2) = A \).

A trace \( \alpha \) is a finite sequence of timed-events \( \alpha \in (\mathbb{R} \times \mathcal{Ev})^* \), representing the execution of a protocol.

**Agents’ Knowledge:** As the trace evolves, agents may gain knowledge by receiving messages from other agents. At the beginning of a protocol execution, every agent is provided with an initial knowledge consisting of all agents’ names and constants, his own nonces and secret keys, and all public keys. We use the function \( \text{init}: \text{Agent} \to \mathcal{P}(\text{Msg}) \) to represent the initial knowledge of an agent:

\[
\text{init}(A) = \text{Agent} \cup \text{Const} \cup \text{Nonce}_A \cup \{sk(A)\} \cup \{pk(B) | B \in \text{Agent}\} \cup \{sh(A, B) | B \in \text{Agent}\},
\]

where \( \text{Nonce}_A \) denotes the set of nonces for a given agent \( A \in \text{Agent} \). We assume that \( \{\text{Nonce}_A | A \in \text{Agent}\} \) forms a partition of the set \( \text{Nonce} \).

Given an agent \( A \) and a trace \( \alpha \), \( dm_A(\alpha) \) denotes the set of all deducible messages from a trace \( \alpha \). This set is inductively defined by the rules in Figure 3.2.

**Network and Intruder:** For a given protocol \( \mathcal{P} \), the set of possible traces \( \text{Tr}(\mathcal{P}) \) is inductively defined by the Start rule (Start), the Intruder rule (Int), the Network rule (Net) and the rules specifying the protocol. The Start, Intruder and Network rules are depicted in Figure 4.3.4.4.

The rules make use of the function \( \text{max}: (\mathbb{R} \times \mathcal{Ev})^* \to \mathbb{R} \), defined as \( \text{max}(\alpha) = \max_{(t,e) \in \alpha} \{t\} \), yields the latest time at which an event of \( \alpha \) occurred. The expression \( d(A, B) \) gives the distance between two agents \( A \) and \( B \) based on an uninterpreted function \( l: \text{Agent} \to \mathbb{R}^3 \), which associates each agent to a location in the real coordinate space \( \mathbb{R}^3 \). It is worth remarking that this interpretation of location assumes that agents are static, including dishonest agents.
\[ \alpha \in \text{Tr}(P) \quad \beta \in \text{Dishonest} \quad t \geq \max(t(\alpha)) \quad m \in \text{dm}(\alpha) \]
\[ \alpha \cdot (t, \text{send}_A(m)[s]) \in \text{Tr}(P) \quad \text{Int} \]
\[ \alpha \in \text{Tr}(P) \quad \beta \in \text{Dishonest} \quad t \geq t' + d(A,B)/c \]
\[ \alpha \cdot (t, \text{recv}_B(m)) \in \text{Tr}(P) \quad \text{Net} \]

Fig. 4. Start, Intruder and Network rules.

The Start rule states that the empty trace \( \epsilon \) is always part of the set of traces. The Intruder rule enables a dishonest agent, typically known as the *intruder* or the *adversary*, to inject (by sending) on the network any of his deducible messages. Finally, the Network rule establishes that a message \( m \) sent by and agent \( A \) can be received by an agent \( B \) without violating a time/location constraint that we describe in the next paragraph. This constraint is actually what makes this model particularly different from standard security models.

The Network rule also enforces that a message sent by an agent \( A \) and received by an agent \( B \) at times \( t' \) and \( t \), respectively, must satisfy \( d(A,B) \leq (t - t') \cdot c \). In this way the physical law that messages cannot travel faster than the speed of light is made explicit. Observe that message loss is captured by not applying the network rule for a given sending event.

**Protocol Specification:** A protocol is specified by a set of rules similar to the rules in Figure 4. Two syntactic restrictions (whose semantic interpretations will be given in Section IV-A) are applied:

- Neither the premises nor the conclusion of a protocol rule contain references to dishonest agents. This means that the behavior of dishonest agents is fully specified by the intruder rule.
- The premise of a protocol rule cannot contain events whose actors are not the same as the actor of the event in the premise of the rule. That is to say, agents are unaware of what other agents do. They can interact exclusively through the network rule.

**Example 1** (Hancke and Kuhn’s protocol), Figure 5 shows the formalization of Hancke and Kuhn’s protocol [20] (see the representation in Figure 2). The first four rules in Figure 5 correspond to the four transmissions that take place in the protocol. The receiving events are derived from the network rule. The last rule from Figure 5 refers to the claim event for the property secure distance-bounding represented as “P is close” in Figure 2.

The function \( \text{used} \): (\( \mathbb{R} \times \text{Ev} \))^* \( \rightarrow \mathcal{P}(\text{Msg}) \) defined as
\[ \text{used}(\alpha) = \bigcup_{(t,e) \in \alpha} \text{sub}(\text{cont}(e)) \]
is utilized to make sure that newly generated nonces are fresh, where \( \text{sub}: \text{Msg} \rightarrow \mathcal{P}(\text{Msg}) \) indicates the set of atomic messages that are sub-

terms of a given message and cont: Ev \( \rightarrow \) Msg gives us the content of a given event. The function subst is recursively defined as follows.

\[
\text{subst}(m) = \begin{cases} 
\text{subst}(m_1) \cup \text{subst}(m_2) & \text{if } m = (m_1, m_2) \\
\text{subst}(m_1) \cup \text{subst}(m_2) & \text{if } m = \{m_1\}_{m_2} \\
\text{subst}(m_1) & \text{if } m = f(m_1) \\
\{m\} & \text{otherwise}
\end{cases}
\]

Example 1 also illustrates the purpose of the information in square brackets at the end of the send actions. In this case, it is implicitly used to define the notion of a session, by extending the behavior of dishonest agents is fully specified by the intruder rule.

**Security Properties:** The model uses claim events as placeholders to indicate where a security property needs to be satisfied. In this paper we focus on the property of *secure distance-bounding*, which is syntactically represented by claims of the form \( \text{claim}_V(P, u, v) \), where \( V, P \in \text{Agent} \) and \( u, v \in \text{Ev} \). A claim event \( \text{claim}_V(P, u, v) \) intuitively means that the agent \( V \) believes that the events \( u \) and \( v \) can be used to correctly compute an upper bound on his distance to \( P \).

As the Intruder rule suggests, dishonest agents might disclose their secret key material by sending them out. This means that two dishonest provers might be indistinguishable to a legitimate verifier. In other words, a verifier \( V \) cannot securely decide whether a particular dishonest prover \( P \) is close, as another dishonest prover \( P' \) could have obtained all \( P \)'s secrets and therefore \( P' \) can impersonate \( P \). This leads to the following statement: \( V \) cannot claim that “\( P \) is close” but \( V \) can claim that “someone who knows \( P \)'s secrets is
\[ \alpha \in \text{Tr}(\mathcal{P}) \quad A \in \text{Honest} \quad \text{Hello}, \text{Hi} \in \text{Const} \quad (t, \text{recv}_A(\text{Hello})) \in \alpha \]
\[
\alpha \cdot (t - 1, \text{send}_A(\text{Hi}) []) \in \text{Tr}(\mathcal{P})
\]

Fig. 6. A protocol rule that leads to incorrect traces.

close”, at most. To capture this notion, we define the relation \( \approx \subseteq \text{Agent} \times \text{Agent} \) as:

\[ \approx = \{ (A, A) \mid A \in \text{Honest} \} \cup \text{Dishonest} \times \text{Dishonest}. \]

We use \( A \neq B \) to indicate that \( (A, B) \notin \approx \). By considering the relation \( \approx \), we provide next a formal definition of secure distance-bounding.

**Definition 1** (Secure distance-bounding). A protocol \( \mathcal{P} \) satisfies secure distance-bounding if and only if:

\[
\forall \alpha \in \text{Tr}(\mathcal{P}), V, P \in \text{Agent}, u, v, w \in \text{Ev}, tw \in \mathbb{R}:
\]

\[
(tw, w) \in \alpha \land w = \text{claim}_V(P, u, v) \implies \exists tu, tv \in \mathbb{R}, P' \in \text{actor}(\alpha) :
\]

\[
(tu, u) \in \alpha \land (tv, v) \in \alpha \land P \approx P' \land d(V, P') \leq \frac{c}{2} (tv - tu).
\]

A distance-bounding protocol is secure if the occurrence of a claim event \( \text{claim}_V(P, u, v) \) in a protocol execution implies that \( V \) has correctly computed an upper bound on his distance to either \( P \) (if \( P \) is honest) or some dishonest agent \( P' \) (if \( P \) is dishonest).

Our definition of secure distance-bounding slightly differs from the original one provided by Basin et al., but the difference is merely notational, allowing us to cleanly formulate our main result in Section V. Note that claim events are formulated in such a way that they relate to a single challenge/response pair. Thus, similar to Basin et al’s approach, we will need to include several claim events if the fast phase cannot be abstracted to a single challenge/response pair.

**IV. The Semantic Domain**

An important characteristic of Basin et al’s approach, as presented in the previous section, is that security protocols are specified using the same type of derivation rules as used for the definition of the general semantics of the system. Consequently, protocol specifications are much more liberal than in comparable formal approaches that define a domain specific language for the definition of protocols. Alternative approaches, like the one by Cremers and Mauw [18] provide a dedicated protocol specification language and impose syntactical or semantical constraints to prevent users from specifying meaningless or simply undesired protocols.

An example of a protocol rule that may be considered undesirable is the one in Figure 6. It specifies that after reception of the message Hello at time \( t \), agent \( A \) sends a message Hi back at time \( t - 1 \). This is clearly an infringement of a time consistency property, because it leads to the trace \((1, \text{recv}_A(\text{Hello})) \cdot (0, \text{send}_A(\text{Hi}) [])\).

The solution proposed by Basin et al. is to consider only those traces that have non-decreasing timestamps for subsequent events. In our approach we will take this line of reasoning one step further, in that we will define a number of assumptions that a proper semantics should satisfy and that are sufficient to derive our main result. We will argue that these properties are valid for the semantics from the previous section, under the assumption of a class of “reasonable” protocol specifications.

**A. Basic Properties of the Semantics**

In line with the previous example, the first property that we formulate is *time consistency*. It states that events of a trace are timestamped in non-decreasing order.

**Property 1** (Time consistency). A protocol \( \mathcal{P} \) satisfies time consistency if for every trace \( \alpha = (t_1, e_1) \cdots (t_n, e_n) \in \text{Tr}(\mathcal{P}) \), it holds that \( t_1 \leq \cdots \leq t_n \).

The second property that we consider is *speed-of-light consistency*. It states that all traces satisfy the restrictions of the speed of light. In particular, this means that the time between the sending of a message by agent \( A \) and the reception of this message by agent \( B \) must be equal to or larger than the distance between the two agents divided by the speed of light.

Because this definition requires the correspondence between a send event and its related receive event, we define the relation \( \sim \subseteq \text{Send} \times \text{Recv} \) as follows:

\[
\sim = \{ (e, e') \in \text{Send} \times \text{Recv} \mid \text{cont}(e) = \text{cont}(e') \}.
\]

The relation \( \sim \) defines whether an event \( e' \) is a receive event that could have occurred as consequence of the send event \( e \). As followed from its formulation, \( \sim \) is not a one-to-one relation. This lines up with the fact that it does not need to be the case that there is a unique send event that triggers a given receive event. In the semantics above, the relation \( \sim \) can be easily derived from the application of the Network rule in Figure 4.

**Property 2** (Speed-of-light consistency). A protocol \( \mathcal{P} \) satisfies speed-of-light consistency if for every trace \( \alpha = (t_1, e_1) \cdots (t_n, e_n) \in \text{Tr}(\mathcal{P}) \) the following holds: for all \( j \in \{2, \ldots, n\} \), if \( e_j \in \text{Recv} \), then there exists \( i \in \{1, \ldots, j - 1\} \) such that \( e_i \sim e_j \) and \( t_j - t_i \geq d(e_i, e_j) / c \).

Even though we define Properties 1 and 2 for protocols, we will also use them in relation to traces. Thus we will talk about time consistency and speed-of-light consistency of a given trace, with the obvious interpretation.

The formulation of the remaining properties requires the notion of *untimed* traces, or simply a sequence of (untimed) events. The projection \( \pi(\alpha) \) of a trace \( \alpha = (t_1, e_1) \cdots (t_n, e_n) \in (\mathbb{R} \times \text{Ev})^* \) is the untimed trace \( e_1 \cdots e_n \in \text{Ev}^* \). Likewise, the projection of the set of traces is defined as \( \pi(\text{Tr}(\mathcal{P})) = \{ \pi(\alpha) \mid \alpha \in \text{Tr}(\mathcal{P}) \} \). We say that two traces \( \alpha \) and \( \beta \) are content-wise equal, denoted \( \alpha \sim \alpha' \), if \( \pi(\alpha) = \pi(\beta) \).
The third property states that traces are built inductively by appending events.

**Property 3** (Prefix-closure). A protocol \( \mathcal{P} \) is prefix-closed if for every \( \gamma = \sigma \cdot e \in \pi(\text{Tr}(\mathcal{P})) \), it holds that \( \sigma \in \pi(\text{Tr}(\mathcal{P})) \).

The fourth property expresses that the notion of time is only used for the verifier’s decision-making process on whether the prover passed the protocol or not. Time will not be used to make any other decision during the execution of the protocol (e.g., to take a different branch depending on the time). This means that any trace can be 

**Property 4** (Time-unawareness). A protocol \( \mathcal{P} \) is time-unaware if for every trace \( \alpha \in \text{Tr}(\mathcal{P}) \) the following holds: for all time consistent and speed-of-light consistent traces \( \beta \in (\mathbb{R} \times \text{Ev})^\ast \), \( \alpha \sim \beta \) implies \( \beta \in \text{Tr}(\mathcal{P}) \).

As mentioned in Section III, different agents only interact through the network via sending and receiving events. As a consequence, a non-receive action can only be triggered by the actor agent’s own preceding actions. In addition, a message cannot be transmitted and received at the same time, unless both actions are taken by the same agent. These two intuitions together lead to the swapping-closure property. It states that events of different agents that occurred at the same time can be swapped.

**Property 5** (Swapping-closure). A protocol \( \mathcal{P} \) is swapping-closed if for every \( \alpha, \beta \in (\mathbb{R} \times \text{Ev})^\ast \), every \( e, e' \in \text{Ev} \), and every \( t \in \mathbb{R} \) such that \( \alpha \cdot (t, e) \cdot (t, e') \cdot \beta \in \text{Tr}(\mathcal{P}) \) and \( \text{actor}(e) \neq \text{actor}(e') \) it holds that \( \alpha \cdot (t, e') \cdot (t, e) \cdot \beta \in \text{Tr}(\mathcal{P}) \).

Agents in the model are universally quantified. Therefore, in a given trace we can replace an agent by another and still obtain a valid trace, as long as both agents are either honest or dishonest. An agent substitution is denoted by \( A \rightarrow B \) where \( A \) and \( B \) are agents. Given a message \( m \in \text{Msg} \), \( m[A \rightarrow B] \) represents the substitution of all occurrences in \( m \) of \( A \) by \( B \). We extend substitutions onto events and traces in the obvious way.

**Property 6** (Substitution-closure). A protocol \( \mathcal{P} \) is substitution-closed if for every \( \sigma \in \pi(\text{Tr}(\mathcal{P})) \) and every \( A, B \in \text{Agent} \) such that \( \{A, B\} \subseteq \text{Honest} \) or \( \{A, B\} \subseteq \text{Dishonest} \), it holds that \( \sigma[A \rightarrow B] \in \pi(\text{Tr}(\mathcal{P})) \).

Observe that \( e \rightsquigarrow e' \) implies \( e[A \rightarrow B] \rightsquigarrow e'[A \rightarrow B] \). We say that a protocol is well-formed if it satisfies the six properties mentioned above.

**B. Validity of the Properties**

As stated in the beginning of the current section, the mechanism for specifying protocols is too liberal to ensure the well-formedness properties. Therefore, we use a restricted format for protocol rules inspired by the example specification of Hancke and Kuhn’s protocol from Figure 5. The restricted format is specified by the rule prototype in Figure 7. We additionally require that \( p + q > 0 \), \( A = \text{actor}(e) = \text{actor}(e_1) = \text{actor}(e_2) = \cdots = \text{actor}(e_q) \).

\[
\begin{align*}
\alpha \in \text{Tr}(\mathcal{P}) & \quad A \in \text{Honest} & & t \geq \text{maxt}(\alpha) \\
(\text{prem}_1, \text{prem}_2, \cdots, \text{prem}_p) & & & \\
(t_1, e_1) \in \alpha & (t_2, e_2) \in \alpha & \cdots & (t_q, e_q) \in \alpha \\
\alpha \cdot (t, e) \in \text{Tr}(\mathcal{P}) & & & \\
\end{align*}
\]

Fig. 7. Prototype of rules that lead to well-formed protocols.

Together with the Start, Intruder and Network rules from Figure 4, the restricted format implies well-formedness of the specified protocol. We will briefly argue the validity of the properties under this restricted format. Time consistency follows from the precondition \( t \geq \text{maxt}(\alpha) \) in the Intruder and Network rules and in the restricted protocol rule. Speed-of-light consistency follows from the precondition \( t \geq t' + d(A, B) / c \) in the Network rule and the requirement that \( e \notin \text{Recv} \) in the restricted protocol rule. Prefix-closure follows from the precondition \( \alpha \in \text{Tr}(\mathcal{P}) \) in all rules, together with the fact that the conclusion extends this trace with a single event. Time-unawareness follows from the fact that in the construction of the traces any time \( t \geq \text{maxt}(\alpha) \) is allowed for the next event, as long as speed-of-light consistency is satisfied. Swapping-closure follows from the premises not involving the timestamps and the expanded event’s actor being the same as the actor of the events in the premises. Substitution-closure expresses the (implicit) universal quantification over agents’ names in all rules.

**V. CAUSALITY-BASED VERIFICATION**

Given the definitions and properties from the previous sections, we can now formulate the notion of causality-based secure distance-bounding and prove that it is equivalent to the original definition of secure distance-bounding from Definition 1. The main feature of this new formulation is that it is causality-based, i.e., it only takes into account the relative occurrence of events, while ignoring the actual timestamps of the events and agents’ locations.

This new formulation strongly relates to authentication properties, such as aliveness (see [18]). It states that for every claim that prover \( P \) is in the vicinity of verifier \( V \), due to a challenge event \( u \) and the reception of its corresponding response event \( v \) in the fast phase, agent \( P \) (or a conspiring agent, if \( P \) is dishonest) must have been active in between these two events. The main difference with Definition 1 is that we require the prover to be active, instead of measuring the time between \( u \) and \( v \).

**Definition 2** (Causality-based secure distance-bounding). A well-formed protocol \( \mathcal{P} \) satisfies causality-based secure...
distance-bounding if and only if:

\[
\forall \sigma \in \pi(\text{Tr}(P)), V, P \in \text{Agent}, u, v \in V:\n
\text{claim}_V(P, u, v) \in \sigma \implies \exists i, j, k \in \{1, \ldots, |\sigma|\}:
\]

\[
i < j < k \land u = \sigma_i, v = \sigma_k \land P \approx \text{actor}(\sigma_j). \quad (2)
\]

In Definition 2 we formalize our causality-based notion of secure distance-bounding. This formulation impacts only the security analysis in the design stage. It does not affect the runtime behavior of the agents executing the protocol. In particular, the verifying agent still has to measure the round-trip time of the message exchanges in the fast phase.

In the remainder of this section, we develop the proof that the causality-based definition is equivalent to the secure distance-bounding property from Definition 1. To do so, we first present two lemmas that follow from the basic properties of the semantic domain described in Section IV-A. They will prove useful when deriving our main result.

Given two events \(e, e' \in E\), we use \(d(e, e') / c\) as a shorthand notation for \(d(\text{actor}(e), \text{actor}(e')) / c\). Also, we say that two timed-events \((t, e), (t', e') \in \mathbb{R} \times E\) satisfy the time/location constraint if \(|t' - t| \geq d(e, e') / c\). For example, all pairs of events used in the network rule satisfy this constraint. We use \(|.|\) to denote the length of a (timed or not) trace, in terms of the number of events.

Next lemma states that a trace can be re-ordered and re-timed so that an event that could not have triggered (directly or indirectly) a later event can be delayed until such later event. Delaying the occurrence of an event must delay the occurrence of its dependent events too.

To develop this result, for every non-empty sequence \(\beta = (t_1, e_1) \cdots (t_n, e_n) \in (\mathbb{R} \times E)^*\) and every \(\delta \in \mathbb{R}\), we denote by \(\langle \beta \rangle_\delta\) the sequence \((t_1 + \delta, e_1) \cdots (t_n + \delta, e_n)\). Furthermore, for every \(S = \{s_1, \ldots, s_{|S|}\} \subseteq \{1, \ldots, n\}\) with \(s_i < s_{i+1}\) for all natural numbers \(i < |S|\), we denote by \(\beta_S\) the sub-sequence \((t_{s_1}, e_{s_1}) \cdots (t_{s_{|S|}}, e_{s_{|S|}})\). Finally, for every \(S \subseteq \{1, \ldots, n\}\), we call the pair \((\beta_S, \beta_{1, \ldots, n} \setminus S)\) a sequence partition of \(\beta\).

**Lemma 1.** Let \(P\) be a well-formed protocol. Then, for every \(\alpha, \beta \in (\mathbb{R} \times E)^*\) and every \((t, e), (t', e') \in \mathbb{R} \times E\) such that \(\alpha \cdot (t, e), \beta \in \text{Tr}(P)\) and \(|\beta| > 0\), there exist a sequence partition \((\beta', \beta'')\) of \(\beta\) and \(\delta \in \mathbb{R}_{\geq 0}\) such that:

- \(\alpha \cdot \beta' \cdot ((t, e), \beta'' \rangle \in \text{Tr}(P)\),
- \(\forall (t', e') \in \beta': (t', e') \in \beta' \iff |t' - t| < d(e, e') / c\).

**Proof:** We proceed by induction on \(|\beta|\). Let \(T \geq 2\) be a natural number.

**Base case:** For every \(\alpha \in (\mathbb{R} \times E)^*\) and every \((t, e), (t', e') \in \mathbb{R} \times E\) such that \(\alpha \cdot (t, e), (t', e') \in \text{Tr}(P)\) it holds that if \(|t' - t| < d(e, e') / c\) then \(\alpha \cdot (t, e), (t', e') \cdot ((t, e), (t', e')) \in \text{Tr}(P)\) for some \(\delta \in \mathbb{R}_{\geq 0}\).

In effect, let \(\alpha \in (\mathbb{R} \times E)^*\) and \((t, e), (t', e') \in \mathbb{R} \times E\) such that \(\tau = \alpha \cdot (t, e), (t', e') \in \text{Tr}(P)\) and \(t' - t < d(e, e') / c\). Let \(\tau' = \alpha \cdot (t', e') \cdot ((t, e), (t', e'))\), which is time and speed-of-light consistent, because if \(e' \in \text{Recv}\) then from \(t' - t < d(e, e') / c\) and Property 2 it follows that \(\exists (t'', e'') \in \alpha: e'' \approx e' \land t - t'' \geq d(e'', e') / c\). Thus, given that \(\tau \sim \tau'\), from Property 4 it follows that \(\tau' \in \text{Tr}(P)\). In addition, \(0 \leq t' - t < d(e, e') / c\) implies that \(\text{actor}(e) \neq \text{actor}(e')\). Therefore, from Property 5 we derive the expected result \(\alpha \cdot (t', e') \cdot ((t, e), (t', e')) \in \text{Tr}(P)\).

**Hypothesis:** For every \(\alpha, \beta \in (\mathbb{R} \times E)^*\) and every \((t, e), (t', e') \in \mathbb{R} \times E\) such that \(\alpha \cdot (t, e), \beta \in \text{Tr}(P)\) and \(|\beta| < T\) there exist a sequence partition \((\beta', \beta'')\) of \(\beta\) and \(\delta \in \mathbb{R}_{\geq 0}\) such that the conditions of the lemma hold.

**Thesis:** For every \(\alpha, \beta \in (\mathbb{R} \times E)^*\) and every \((t, e) \in \mathbb{R} \times E\) such that \(\alpha \cdot (t, e), \beta \in \text{Tr}(P)\) and \(|\beta| = T\) there exist a sequence partition \((\beta', \beta'')\) of \(\beta\) and \(\delta \in \mathbb{R}_{\geq 0}\) such that the conditions of the lemma hold.

In effect, let \(\alpha, \beta \in (\mathbb{R} \times E)^*\) and \((t, e) \in \mathbb{R} \times E\) such that \(\alpha \cdot (t, e), \beta \in \text{Tr}(P)\). If \(\forall (t', e') \in \beta: t' - t < d(e, e') / c\) then after \(T\) swaps of \(e\) rightwards (as we did in the base case) we obtain that \(\alpha \cdot \beta \cdot ((t, e), _{\text{max}}(\beta), t) \in \alpha\).

So, let \(a, b \in (\mathbb{R} \times E)^*\) and \((t, e) \in \mathbb{R} \times E\) such that \(\beta = \alpha \cdot (t', e') \cdot b\) and \(t' - t < d(e, e') / c\) and \(\forall (t', e') \in a: t'' - t \geq d(e, e') / c\).

Let \(\tau = \alpha \cdot (t, e), \beta \cdot (t', e') \in \mathbb{R} \times E\) and \(a' \in (\mathbb{R} \times E)^*\) such that \((t, e), a' = (t', e')\). Therefore, \(\tau = \alpha \cdot a \cdot (t', e') \cdot (t', e') \cdot b\). Hence,

\[
\tau' = \alpha \cdot a \cdot (t' + t'' - t), (t', e') \cdot (t', e') \cdot (b) \in \text{Tr}(P).
\]

is time and speed-of-light consistent. Thus, from Property 4 and the fact that \(\tau \sim \tau'\) it follows that \(\tau' \in \text{Tr}(P)\). But, \(t' - t \geq d(e, e') / c\) and \(t' - t < d(e, e') / c\) and the triangle inequality give us:

\[
t' - t'' < d(e, e') / c - d(e, e') / c < d(e'', e') / c,
\]

which implies that \(\text{actor}(e'') \neq \text{actor}(e')\). From this result and Property 5 and given that \(\tau' \in \text{Tr}(P)\) we obtain:

\[
\tau'' = \alpha \cdot a' \cdot \tau' \cdot (t', e') \cdot (b) \in \text{Tr}(P).
\]

Thus, by repeating this process of moving \(e\) leftwards, after \(a'\) more swaps in \(\tau''\) we obtain that:

\[
\omega = \alpha \cdot (t', e') \cdot (\varphi') \cdot (t, e) \cdot (\varphi'') \in \text{Tr}(P).
\]

and, after canceling out the \(\delta\)’s:

\[
\forall (t', e') \in \varphi:
\]

\[
(t', e') \in \varphi' \iff t' - t < d(e, e') / c. \quad (3)
\]

Given that all timestamps in \(\omega\) are shifted in a monotonically non-decreasing way with respect to \(\tau\) we get that:

\[
\omega' = \alpha \cdot (t', e') \cdot (\varphi') \cdot (t, e) \cdot (\varphi'') \in \text{Tr}(P),
\]

is time speed-of-light consistent. Thus, from Property 4 and the fact that \(\omega \sim \omega'\) it follows that \(\omega' \in \text{Tr}(P)\). Furthermore,
given that all timestamps in $\omega'$ are shifted in a monotonically non-decreasing way with respect to $\omega$, we deduce that:

$$\omega'' = \alpha \cdot (t', e') \cdot \varphi' \cdot ((t, e) \cdot \varphi'')_{\mu'}$$

is also time and speed-of-light consistent. Thus, from Property 4 and the fact that $\omega' \sim \omega''$ it follows that $\omega'' \in \text{Tr}(\mathcal{P})$. This result together with Equation 3 complete the proof. ■

The second lemma of this section concerns agent substitutions. We extend Property 6 from the set of untimed-traces $\pi(\text{Tr}(\mathcal{P}))$ of a given protocol $\mathcal{P}$ to the set of timed-traces $\text{Tr}(\mathcal{P})$. The lemma proves that, given a protocol's valid trace $\alpha = (t_1, e_1) \cdots (t_n, e_n)$, it is possible to replace an agent $A$ by another agent $B$ (under certain conditions described in the lemma) to obtain another valid trace $\alpha' = (t'_1, e'_1) \cdots (t'_n, e'_n)$ such that the difference between $t'_i$ and $t_i$ only depends on the number of events before the $i$-th event on $\alpha$ that were executed by $A$. Consequently, the time-difference between two events of $\alpha$ where $A$ does not act is equal to the time-difference between the corresponding events of $\alpha'$. This is actually a strong result because it implicitly shows that event-intervals where the prover does not act cannot be used to securely upper-bound the prover-to-verifier distance.

**Lemma 2.** Let $\mathcal{P}$ be a well-formed protocol and $\alpha = (t_1, e_1) \cdots (t_n, e_n) \in \text{Tr}(\mathcal{P})$. Let $A \in \text{actor}(\alpha)$, $B \in \text{Agent} \setminus \text{actor}(\alpha)$ such that either $\{A, B\} \subseteq \text{Honest}$ or $\{A, B\} \subseteq \text{Dishonest}$. Then there exists $\mu \in \mathbb{R}_{\geq 0}$ such that $\alpha' = (t'_1, e'_1) \cdots (t'_n, e'_n) \in \text{Tr}(\mathcal{P})$ where for all $i \in \{1, \ldots, n\}$ it holds that:

$$e'_i = e_i[A \rightarrow B]$$

and

$$t'_i = t_i + \mu \cdot q_i,$$

where

$$q_i = |\{j \in \{1, \ldots, i - 1\} \setminus \text{actor}(e_j) = A\}| + s_i, and$$

$$s_i = 1 \text{ if } (A = \text{actor}(e_i) \land e_i \in \text{Recv}), or \text{ otherwise.} s_i = 0.$$

**Proof:** Consider the set $R = \{B\} \cup \text{actor}(\alpha)$ and $\mu = \max(d(A, X)/c)$. We will prove that $\alpha' = (t'_1, e'_1) \cdots (t'_n, e'_n) \in \text{Tr}(\mathcal{P})$. To do so we will first prove time and speed-of-light consistency for $\alpha'$.

**Time consistency:** For all $i \in \{1, \ldots, n - 1\}$, we have that $q_{i+1} \geq q_i$ and therefore $t'_{i+1} - t'_i = t_{i+1} - t_i + \mu \cdot (q_{i+1} - q_i) \geq t_{i+1} - t_i \geq 0$.

**Speed-of-light consistency:** Let $j \in \{1, \ldots, n\}$ such that $e_j \in \text{Recv}$.

Also, as $\alpha$ is speed-of-light consistent, we derive that there exists $i < j$ such that $e_j \sim e_i$ and $t_j - t_i \geq d(e_i, e_j)/c$. Hence, given that $e'_i \sim e'_j$, it becomes sufficient to prove that $t'_j - t'_i \geq d(e'_i, e'_j)/c$. Let us consider the three cases:

1. $A = \text{actor}(e_i)$. In this case $q_j \geq q_i + 1$ because $e_i \notin \text{Recv}$. Therefore $t'_j - t'_i \geq t_j - t_i + \mu \cdot d(e'_i, e'_j)/c$ as $\mu \geq d(e'_i, e'_j)/c$.

2. $A \neq \text{actor}(e_i)$ and $A = \text{actor}(e_j)$. In this case we have again $q_j \geq q_i + 1$ as $e_j \in \text{Recv}$, and it follows analogously to the previous case.

3. $A \notin \{\text{actor}(e_i), \text{actor}(e_j)\}$. This case gives us $\text{actor}(e_i) = \text{actor}(e'_i)$ and $\text{actor}(e_j) = \text{actor}(e_j)$. Thus, $d(e_i, e_j)/c = d(e'_i, e'_j)/c$ and therefore $t'_j - t'_i = t_j - t_i + \mu \cdot (q_j - q_i) \geq t_j - t_i \geq d(e_i, e_j)/c = d(e'_i, e'_j)/c$.

Thus, $\alpha'$ is time consistent and speed-of-light consistent. Consider now $\sigma = \pi(\alpha)$. From Property 6 we have that $\sigma[A \rightarrow B] \in \pi(\text{Tr}(\mathcal{P}))$. Therefore, there exists $\gamma \in \text{Tr}(\mathcal{P})$ such that $\pi(\gamma) = \pi[A \rightarrow B]$. Finally, given that $\gamma \sim \alpha'$, from Property 4 we obtain $\alpha' \in \text{Tr}(\mathcal{P})$. ■

**Theorem 1.** A well-formed protocol $\mathcal{P}$ satisfies secure distance-bounding (Definition 1) if and only if $\mathcal{P}$ satisfies causality-based secure distance-bounding (Definition 2).

**Proof:** We will proceed by proving Sufficiency (i.e., Equation 1 $\Rightarrow$ Equation 2) and Necessity (i.e., Equation 2 $\Rightarrow$ Equation 1):

**Sufficiency:** Assume Equation 1 holds and Equation 2 does not. Our goal is to reach a contradiction. The statement that Equation 2 does not hold is equivalent to stating that there exist $\sigma = \sigma_1 \cdots \sigma_n \in \pi(\text{Tr}(\mathcal{P}))$, $V, P \in \text{Agent}$, $u, v \in \text{Ev}$ and $l \in \{1, \ldots, n\}$ such that $\sigma_1 = \text{claim}_v (P, u, v)$ and:

$$\forall i, j, k \in \{1, \ldots, n\}:
\quad u = \sigma_1 \land v = \sigma_k \land i < j < k \implies P \neq \text{actor}(\sigma_j). \quad (4)$$

Consider now the following sets:

$$IK = \{(i, k) \in \mathbb{N} \times \mathbb{N} \mid \sigma_i = u \land \sigma_k = v\},$$

$$J = \{j \in \mathbb{N} \mid \exists (i, k) \in IK: i < j < k\},$$

$$(G_1, \ldots, G_g) = \{G \in \text{actor}(\sigma) \mid P \approx G\}.$$

If $P$ is honest, then the set $\{G_1, \ldots, G_g\}$ consists of all dishonest agents acting in $\sigma$.

Let $\text{Eve}, \text{Charlie} \in \text{Agent} \setminus \text{actor}(\sigma)$ be two different agents such that $\{P, \text{Eve}, \text{Charlie}\} \subseteq \text{Honest}$ or $\{P, \text{Eve}, \text{Charlie}\} \subseteq \text{Dishonest}$.

Consider the sequence of traces $\sigma_1, \ldots, \sigma_{g+1} \in \pi(\text{Tr}(\mathcal{P}))$ such that $\sigma_1 = \sigma$ and for all $i \in \{1, \ldots, g\}$, $\sigma_{i+1} = \sigma[G_i \rightarrow \text{Eve}]$. The fact that $\sigma_1, \ldots, \sigma_{g+1} \in \pi(\text{Tr}(\mathcal{P}))$ follows from the substitution-closedness property. Hence, let $e_1 \cdots e_n = \sigma_{g+1}$, i.e., the trace resulting from $\sigma$ after the successive substitutions of all agents $G_1, \ldots, G_g$ by $\text{Eve}$. Therefore $\mathcal{N} \in \text{Agent}$ exists such that:

$$\text{actor}(e_1 \cdots e_n) = \{V, \text{Eve}\} \cup \mathcal{N} and$$

$$\forall E \in \mathcal{N}: E \neq E. \quad (5)$$

Let $t_1, \ldots, t_n \in \mathbb{R}$ such that $(t_1, e_1) \cdots (t_n, e_n) \in \text{Tr}(\mathcal{P})$. Observe that the $t_i$’s exist because $e_1 \cdots e_n \in \pi(\text{Tr}(\mathcal{P}))$.

Hence, from Equations 1 and 5 and given that $e_i = \text{claim}_v (\text{Eve}, e_i, c_k)$ for some $(i, k) \in IK$, we derive that $\delta \in \mathbb{R}_{\geq 0}$ exists such that:

$$d(V, \text{Eve}) + \delta = \frac{\varepsilon}{2} \max_{(i, k) \in IK} \{t_k - t_i\}. \quad (6)$$

From Lemma 2 we have that there exist $\mu \in \mathbb{R}_{\geq 0}$, $(t_1', e_1') \cdots (t_n', e_n') \in \text{Tr}(\mathcal{P})$ and $q_1, \ldots, q_n \in \mathbb{N}$ such that for all $i \in \{1, \ldots, n\}$, $e_i = e_i[\text{Eve} \rightarrow \text{Charlie}]$ and $t'_i = t_i + \mu \cdot q_i$. (see the construction of the $q_i$'s in Lemma 2).
the other hand, from Equation 4 we have that \( \forall j \in J: \text{Eve} \neq \text{actor} (e_j) \). Therefore

\[
\forall (i,k) \in IK: t'_k - t'_i = t_k - t_i. \tag{7}
\]

Furthermore, given that \( \{\text{Eve}, \text{Charlie}\} \subseteq \text{Honest} \) or \( \{\text{Eve}, \text{Charlie}\} \subseteq \text{Dishonest} \), it holds that:

\[
\text{actor} (e'_1 \cdots e'_n) = \{V, \text{Charlie}\} \cup N \quad \text{and} \quad \forall C \in N: \text{Charlie} \neq C. \tag{8}
\]

Again, \( e'_i = \text{claim}_V (\text{Charlie}, e'_i, e'_k) \) for some \( (i,k) \in IK \), so from Equations 1 and 8 we derive:

\[
d(V, \text{Charlie}) \leq \frac{c}{2} \max_{(i,k) \in IK} \{t'_k - t'_i\}. \tag{9}
\]

Finally, from Equations 6, 7 and 9 we derive that \( d(V, \text{Charlie}) \leq d(V, \text{Eve}) + \delta \). This is a contradiction, as \( \delta \) does not depend on \( \text{Charlie} \) who is an arbitrary agent from the same set as \( P \) in Honest or Dishonest. Therefore we can always find \( \text{Charlie} \) such that his distance to \( V \) is larger than \( d(V, \text{Eve}) + \delta \).

**Necessity:** Assume Equation 2 holds. We will prove that Equation 1 holds as well.

In effect, let \( (t_1, e_1) \cdots (t_n, e_n) \in \text{Tr} (P) \), \( l \in \{1, \ldots, n\} \), \( V, P \in \text{Agent} \), and \( u, v \in \text{Ev} \) such that \( e_i = \text{claim}_V (P, u, v) \). In addition, let \( i \in \{1, \ldots, n\} \) be the smallest number such that \( e_i = u \) and \( k \in \{i+1, \ldots, n\} \) be the largest number such that \( e_k = v \), and \( J \) be the set of all \( j \in \{i+1, \ldots, k-1\} \) such that \( P \approx \text{actor} (e_j) \). From Equation 2 it follows that \( i \) and \( k \) exist, and \( J \neq \emptyset \).

From Lemma 1 we have that there exist a sequence partition \((\beta', \beta'')\) of \( \beta = (t_{i+1}, e_{i+1}) \cdots (t_n, e_n) \) and \( \delta \in \mathbb{R}_{\geq 0} \) such that

\[
\phi = \alpha \cdot \beta' \cdot \langle \langle t_i, e_i \rangle, \beta'' \rangle_{\delta} \in \text{Tr} (P) \quad \text{where} \quad \alpha = (t_1, e_1) \cdots (t_{i-1}, e_{i-1})
\]

\[
\forall (t', e') \in \beta: (t', e') \in \beta' \leftrightarrow t' - t_i < d(e_i, e') / c. \tag{10}
\]

Furthermore, let \( j \) be the smallest number from \( J \) such that \( (t_j, e_j) \in \beta'' \). Notice that \( j \) exists due to Equation 2 and that \( \phi \in \text{Tr} (P) \). Hence, from Equation 10 we derive:

\[
t_j - t_i \geq d(e_i, e_j) / c. \tag{11}
\]

From \( (t_j, e_j) \in \beta'' \) it follows also that \( \phi \) can be written as \( \phi = \alpha \cdot \beta' \cdot \langle \langle t_i, e_i \rangle, \beta'' \rangle_{\delta} \cdot \langle \langle t_j, e_j \rangle, \beta'' \rangle_{\delta} \cdot \langle \langle t_j, e_j \rangle, \beta'' \rangle_{\delta} \beta'_{\delta} \in \text{Tr} (P) \) (12) and, after canceling out the \( \delta'\)s:

\[
\forall (t', e') \in b: (t', e') \in b' \leftrightarrow t' - t_j < d(e_j, e') / c. \tag{13}
\]

Hence, from Equations 2 and 12 and given that \( j \) is minimal we deduce that \( (t_k, e_k) \in \beta'' \). Thus, Equation 13 gives us:

\[
t_k - t_j \geq d(e_j, e_k) / c. \tag{14}
\]

Finally, Equations 11 and 14 give us the expected result \( t_k - t_i \geq d(e_i, e_j) / c + d(e_j, e_k) / c = 2d(e_i, e_j) / c \).

---

1 [https://github.com/orgetp/dbverify]

2 No attack succeeds with non-negligible probability.
If all nonces \( \alpha, \beta \in \{0,1\}^{2n} \) are correct, then \( \text{Out}_V = 1 \); else \( \text{Out}_V = 0 \)

By requesting \( \alpha, \beta, \sigma \) \( \{0,1\}^{2n} \) from \( V \), an intruder correctly replying to the challenges with 
\[ f(c, m, \alpha, \beta) \] 
\( \text{Out}_V \) follows. An intruder follows the attack above and depicted in Fig. 9. A mafia fraud attack does not work. The reason is that the intruder does not know the secret key shared between \( P \) and \( V \). Thus the intruder is prevented from re-encrypting the message received from \( P \) with the correct key. Nevertheless, a distance-hijacking type of attack exists irrespective of the encryption scheme. The attack is represented in Figure 11. Assume an honest prover \( P \) is close to the verifier \( V \), while the intruder \( I \) is far from \( V \). As before, \( P \) executes the protocol to prove its proximity to \( I \). This allows \( I \) to learn \( \alpha, \beta \). Thus \( I \) starts a session with \( V \) by using the nonces \( \alpha, \beta \) from \( P \). At this point, \( V \) believes \( I \) is a legitimate prover and accepts its signature. During the fast phase, \( P \), which is close to \( V \), receives the challenge (supposedly from \( I \)) sent by \( V \) and replies correctly. Then \( V \) receives the response \( f(c, m, \alpha, \beta) \) (supposedly from \( I \)) from \( P \) who is close to \( V \), and finishes the protocol with \( I \) correctly.

Neither of the two described attacks are possible when considering the adversary model used by the authors of the TREAD protocol, because their model does not allow for “malicious” verifiers. In their model an honest prover will fail to initiate a communication with an untrusted verifier as the first message in each attack will not be sent. This adversary model is weaker than other models that are more common in the distance-bounding literature.

B. Verification Results and Discussion

We applied the above analysis methodology on a number of distance-bounding protocols. To the best of our knowledge, this is the first large-scale automated security analysis of distance-bounding protocols in literature.

Table I summarizes the results of our analysis. The columns Code and Time refer to the code complexity (number of lines of code) and the analysis runtime (in seconds), respectively.

To measure runtime, we ran the analysis 10 times for each
protocol and computed the average time. The column Attacks indicates the type of attack found (if any) by Tamarin: mafia fraud (MF), distance fraud (DF), or distance hijacking (DH). The protocols were analysed using a 64-bit Ubuntu 16.04 LTS computer with 15.5 Gb of RAM memory and a processor Intel Core i7-6700HQ CPU @ 2.60GHz × 8.

We remark that the Tree-based, Poulidor, Hancke and Kuhn’s and Uniform protocols have equivalent Tamarin implementation as their symbolic formalization is the same. Similarly, the Brands and Chaum (BC) protocol versions with Fiat-Shamir and Schnorr identification schemes have also the same representation. When analysing these two versions of the protocol, we found a distance-fraud attack against them. However, as the authors have pointed out, such an attack is no longer possible if a challenge/response causal relation is used during the fast phase, such as the XOR operation employed in the signature-based version of the protocol.

On average, the Tamarin implementation of a protocol consists of 194 lines of code and the analysis takes 5.62 seconds. A total of 5 protocols (and their variations) were found vulnerable to attacks, of which 3 had been already reported flawed in the literature. The two remaining protocols are TREAD (whose analysis was detailed in Section VI-A) and PaySafe [25].

Figure 12 shows a representation of the PaySafe protocol, which is a variant of the classical EMV contactless payment protocol. PaySafe features a distance-bounding mechanism to avoid relay attacks. Although not in contradiction with the authors’ claim regarding PaySafe security, our Tamarin analysis found a successful distance-fraud attack against it. This attack is possible as there is no causal relation between the challenge and response in the fast phase (dashed arrows). Consequently, a dishonest prover $C$ can send $(ATC, n_C)$ before receiving $(UN, amount)$. A simple solution to this attack is to include the nonce $UN$ in the response message.

When considering distance hijacking, our security analysis is consistent with the analysis performed by Cremers et al. in [10]. That is to say, in general protocols based on Brand and Chaum’s design (see Section II, third paragraph) are vulnerable to this type of attacks, whereas those based on Hancke and Kuhn’s are not. In addition, we observed that protocols following Hancke and Kuhn’s approach seem to be resistant not only to distance hijacking but also to mafia and distance frauds.

A few of the considered protocols have been automatically analysed in previous works. Those protocols are Brands and Chaum’s and its variations, as well as Meadows et al’s with $F(\ldots) = (N_V, N_P \oplus P)$, which were analysed in
Isabelle/HOL\(^3\) by using Basin et al’s model [13], [19]. No formal symbolic analysis (automatic or not) has been reported for the rest of the protocols from Table I.

Our method compares well with the Isabelle/HOL implementation of Basin et al. While our approach is fully automatic, proving a protocol insecure with Isabelle/HOL requires user-assistance to prove the existence of an attack trace. In addition, the code complexity of a protocol when implemented in Isabelle/HOL tends to be much larger. For example, the implementation of Brands and Chaum’s protocol consists of 185 lines of Tamarin code, whilst the Isabelle implementation (including attack trace) takes 653.

VII. Conclusion

In this work, we addressed the topic of formal verification of distance-bounding protocols. We described and analysed a tool-supported verification framework by Basin et al [13], [19] based on timed-events and agents’ locations. By considering the language and semantics of this formalism, we characterized a semantic domain of well-formed distance-bounding protocols in which the timestamps associated to the agents’ actions are only utilized for proximity verification purposes and not for, e.g., taking a different branch in the execution. This is not a trivial class of distance-bounding protocols but it contains, to the best of our knowledge, all protocols published to date. Our main result consists of the first causality-based security model for symbolic verification of distance-bounding protocols, which we prove equivalent to Basin et al’s model.

Our proposal does not consider time and location, but is instead based on the order of events in the execution traces. By implementing our proposed model in the Tamarin verification tool, we automatically analysed various state-of-the-art protocols. It is therefore the first fully automated formal verification framework for distance-bounding protocols. With our automated analysis, we identified unreported vulnerabilities in two recent protocols: a mafia-fraud and a distance-hijacking attack on the TREAD protocol [24], and a distance-fraud attack against the EMV-based contactless payment protocol PaySafe [25].

As future work, we plan to extend the formalism to capture terrorist-fraud attacks (which are not covered in Basin et al’s model either), and formalize the different attacks and protocols’ characteristics to prevent them. We also plan to extend our methodology as to capture probabilistic reasoning in a causality-based model. This will allow us to automatically determine the probability of success of a given attack against a distance-bounding protocol.

Acknowledgment

This work was supported by the Luxembourg National Research Fund under the grants AFR-PhD-10188265 and C15-IS-10428112. We thank S. Delaune, A. Debant, I. Boureauau, and T. Chothia for their observations on our manual proofs and Tamarin models.

\(^3\)http://www.infsec.ethz.ch/research/software/protoveriphy.html

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